

MMathPhys/MSc in Mathematical and Theoretical Physics

Michaelmas Courses

Advanced Philosophy of Physics

Department: Philosophy

Lecturers: Prof Adam Caulton, Prof James Read, Prof Christopher Timpson

Course Weight: 1.5 units/24 lectures (continues in Hilary)

Assessment Method: mini-project or homework completion

Course Synopsis: This series of classes will cover contemporary topics in the philosophy of physics, with emphasis on: thermal physics (thermodynamics and statistical mechanics), the role of symmetries in physical theories, spacetime (especially the general theory of relativity), and advanced topics in the philosophy of quantum theory (which may include the role of decoherence in solving the measurement problem, the interpretation of probability, and topics in quantum field theory).

Those MMathPhys and MSc students taking the course in the mini-project or homework mode also receive 8 hours of tutorials (usually as one of a pair) from one of the external lecturers. These tutorials are usually spread throughout the year, with the first 4 in Michaelmas Term. Students are expected to produce an essay of between 2,000-2,500 words for each tutorial. For those taking the course in mini-project mode, two of these tutorials will be given over to discussing drafts of the two 5,000-word essays submitted by each candidate for assessment in week 4 of Trinity Term.

Anyons and Topological Quantum Field Theory

Department: Physics

Lecturer: Prof Steve Simon

Course Weight: 1 unit/ 16 lectures

Assessment Method: written exam in HT week 0 or homework completion

Course Synopsis: The intersection of topology and quantum mechanics is an enormous and still growing field. It touches upon physics topics ranging from quantum gravity to quantum information to materials physics and condensed matter experiment, as well as being one of the most interesting directions in the mathematical study of topology. This is the backdrop upon which we build. A number experiments from the last few years have finally detected and measured anyons — particles that are neither bosons nor fermions — in condensed matter systems (GaAs quantum wells, graphene) as also as in rudimentary quantum computers (superconducting qubits, trapped ion qubits, rydberg atoms). The presence of anyons tells us that our systems are necessarily nontrivial topological quantum field theories! This makes the topic particularly exciting right now!

For a full lecture syllabus, see here: [2024 Anyons Lecture Syllabus](#)

C3.1 Algebraic Topology

Department: Maths

Lecturer: Prof Andras Juhasz

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

General Prerequisites: A3 Rings and Modules is essential, in particular a solid understanding of groups, rings, fields, modules, homomorphisms of modules, kernels and cokernels, and classification of finitely generated abelian groups.

A5 Topology is essential, in particular a solid understanding of topological spaces, connectedness, compactness, and classification of compact surfaces. B3.5 Topology and Groups is helpful but not necessary, in particular the notion of homotopic maps, homotopy equivalences, and fundamental groups will be recalled during the course. There will be little mention of homotopy theory in this course as the focus will be instead on homology and cohomology.

Course Synopsis: Homology theory is a subject that pervades much of modern mathematics. Its basic ideas are used in nearly every branch, pure and applied. In this course, the homology groups of topological spaces are studied. These powerful invariants have many attractive applications. For example we will prove that the dimension of a vector space is a topological invariant and the fact that 'a hairy ball cannot be combed'.

C3.3 Differentiable Manifolds

Department: Maths

Lecturer: Prof Dominic Joyce

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

General Prerequisites: A5: Topology and ASO: Introduction to Manifolds are strongly recommended. (Notions of Hausdorff, open covers, smooth functions on $\mathbb{R}^n$ will be used without further explanation.) Useful but not essential: B3.2 Geometry of Surfaces.

Course Synopsis: A manifold is a space such that small pieces of it look like small pieces of Euclidean space. Thus a smooth surface, the topic of the Geometry of Surfaces course, is an example of a (2-dimensional) manifold. Manifolds are the natural setting for parts of classical applied mathematics such as mechanics, as well as general relativity. They are also central to areas of pure mathematics such as topology and certain aspects of analysis.

In this course we introduce the tools needed to do analysis on manifolds. We prove a very general form of Stokes' Theorem which includes as special cases the classical theorems of Gauss, Green and Stokes. We also introduce the theory of de Rham cohomology, which is central to many arguments in topology.

C3.4 Algebraic Geometry

Lecturer: Prof Damian Rossler

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

General Prerequisites: A3 Rings and Modules and B2.2 Commutative Algebra are essential. Noetherian rings, the Noether normalisation lemma, integrality, the Hilbert Nullstellensatz and dimension theory will play an important role in the course. B3.3 Algebraic Curves is useful but not essential. Projective spaces and homogeneous coordinates will be defined in C3.4, but a working knowledge of them would be useful. There is some overlap of

topics, as B3.3 studies the algebraic geometry of one-dimensional varieties. Courses closely related to C3.4 include C2.2 Homological Algebra, C2.7 Category Theory, C3.7 Elliptic Curves, C2.6 Introduction to Schemes; and partly related to: C3.1 Algebraic Topology, C3.3 Differentiable Manifolds, C3.5 Lie Groups.

Course synopsis: Algebraic geometry is the study of algebraic varieties: an algebraic variety is roughly speaking, a locus defined by polynomial equations. One of the advantages of algebraic geometry is that it is purely algebraically defined and applied to any field, including fields of finite characteristic. It is geometry based on algebra rather than calculus, but over the real or complex numbers it provides a rich source of examples and inspiration to other areas of geometry.

C5.5 Perturbation Methods

Department: Maths

Lecturer: Prof Ruth Baker

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

General Prerequisites: Knowledge of core complex analysis and of core differential equations will be assumed, respectively at the level of the complex analysis in the Part A (Second Year) course Metric Spaces and Complex Analysis and the phase plane section in Part A Differential Equations I. The final section on approximation techniques in Part A Differential Equations II is highly recommended reading if it has not already been covered.

Course Synopsis: Perturbation methods underlie numerous applications of physical applied mathematics: including boundary layers in viscous flow, celestial mechanics, optics, shock waves, reaction-diffusion equations, and nonlinear oscillations. The aims of the course are to give a clear and systematic account of modern perturbation theory and to show how it can be applied to differential equations.

C6.1 Numerical Linear Algebra

Department: Maths

Lecturer: Prof Yuji Nakatsukasa

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

General Prerequisites: Only elementary linear algebra is assumed in this course. The Part A Numerical Analysis course would be helpful, indeed some swift review and extensions of some of the material of that course is included here.

Course Synopsis: Linear Algebra is a central and widely applicable part of mathematics. It is estimated that many (if not most) computers in the world are computing with matrix algorithms at any moment in time whether these be embedded in visualization software in a computer game or calculating prices for some financial option. This course builds on elementary linear algebra and in it we derive, describe and analyse a number of widely used constructive methods (algorithms) for various problems involving matrices.

C7.5 General Relativity I

Department: Maths

Lecturer: Dr Christopher Couzens

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

General Prerequisites: Special Relativity, Classical Mechanics and Electromagnetism

Course Synopsis: The course is intended as an introduction to general relativity, covering both its observational implications and the new insights that it provides into the nature of spacetime and the structure of the universe. Familiarity with special relativity and electromagnetism as covered in the Part A and Part B courses will be assumed. The lectures will review Newtonian gravity, special relativity (from a geometric point of view), and then move on to cover physics in curved space time and the Einstein equations. These will then be used to give an account of planetary motion, the bending of light, the existence and properties of black holes and elementary cosmology.

Groups and Representations

Department: Physics

Lecturer: Prof Andre Lukas

Course Weight: 1.5 units/ 24 lectures

Assessment Method: written exam in HT week 0 and homework completion

Course Synopsis: Modern theories of particle physics are based on symmetry principles and use group theoretical tools extensively. Besides the standard Poincaré/Lorentz invariance of all such theories, one encounters internal (continuous) groups such as $SU(3)$ in QCD, $SU(5)$ and $SO(10)$ in grand unified theories (GUTs), and E_6 and E_8 in string theory. Discrete groups also play an important role in particle physics model building, for example in the context of models for fermion masses.

The main purpose of this course is to develop the understanding of groups and their representations, including finite groups and Lie groups. Emphasis is placed on a mathematically satisfactory exposition as well as on applications to physics and practical methods needed for "routine" calculations.

For a list of prerequisites and suggested reading see here: [Groups and Reps outline](#)

Kinetic Theory

Department: Physics

Lecturers: Prof Alex Schekochihin, Dr Paul Dellar and Dr Robert Ewart

Course Weight: 1.75 units/28 lectures

Assessment Method: written exam in week 0 HT or homework completion

Course Synopsis:

Part I (9 lectures). Kinetic theory of gases. Timescales and length scales. Hamiltonian mechanics of N particles. Liouville's Theorem. Reduced distributions. BBGKY hierarchy. Boltzmann-Grad limit and truncation of BBGKY equation for the 2-particle distribution assuming a short-range potential. Boltzmann's collision operator and its conservation properties. Boltzmann's entropy and the H-theorem. Maxwell-Boltzmann distribution. Linearised collision operator. Model collision operators: the BGK operator, Fokker-Planck operator. Derivation of hydrodynamics via Chapman-Enskog expansion. Viscosity and thermal conductivity.

Part II (10 lectures). Kinetic theory of plasmas and quasiparticles. Kinetic description of a plasma: Debye shielding, micro- vs. macroscopic fields, Vlasov-Maxwell equations. Klimontovich's version of BBGKY (non-examinable). Plasma frequency. Partition of the dynamics into equilibrium and fluctuations. Linear theory: initial-value problem for the Vlasov-Poisson system, Laplace-transform solution, the dielectric function, Landau prescription for calculating velocity integrals, Langmuir waves, Landau damping and kinetic instabilities (driven by beams, streams and bumps on tail), Weibel instability (non-examinable), sound waves, their damping, ion-acoustic instability, ion-Langmuir oscillations. Energy conservation. Heating. Entropy and free energy. Ballistic response and phase mixing. Role of collisions. Elements of kinetic stability theory. Quasilinear theory: general scheme. QLT for bump-on-tail instability in 1D. Introduction to quasiparticle kinetics.

Part III (9 lectures). Kinetic theory of self-gravitating systems. Unshielded nature of gravity and implications for self-

gravitating systems. Virial theorem, negative specific heat and impossibility of thermal equilibrium. Escape, impact of fluctuations. Mean-field approximation, angle-action variables, self-consistent potential, biorthonormal potential-density pairs. Relaxation driven by fluctuations in mean-field. Long-time response to initial perturbation. Fokker-Planck equation. Computation of the diffusion coefficients in terms of resonant interactions. Application to a tepid disc.

For further details see here: [Kinetic Theory course](#)

Quantum Field Theory

Department: Physics

Lecturer: Prof John Wheeler

Course weight: 1.5 units/24 lectures

Assessment method: written exam in 0 HT

Course synopsis:

1. Introduction, and Why do we need quantum field theory?
2. Relativistic wave equations
3. Formalism of classical field theory
4. Canonical quantisation of the real scalar field
5. Charge and complex fields
6. Canonical quantisation of the fermion field
7. Interacting fields, formalism and the perturbation expansion
8. Scattering and decay, their relation to amplitudes
9. Calculation of low order Feynman diagrams
10. Regularization and renormalizable QFTs

Quantum Matter 1: Phases of Matter and Field Theories

Department: Physics

Lecturer: Prof Steve Simon

Course weight: 1 unit/16 lectures

Assessment method: written exam in week 6-8 TT

Course synopsis: This course serves as part of the C6 theory option and also serves as a notional prerequisite for several of the quantum matter courses (QM2,QM3,QM4) that follow.

Part 1: Phases and Phase Transitions. Phase transitions and Universality. Landau Theory and Applications. Ginzburg-Landau theory: Upper and Lower critical dimensions. Spontaneous Symmetry Breaking. Goldstone modes. Mermin Wagner Theorem.

Part 2: Many Body Quantum Field Theory. Working with Fock Space and Second Quantization. Applications to Fermi Systems, Weakly interacting Bosons (Bogoliubov theory) and Spin waves.

Quantum Processes in Hot Plasma

Department: Physics

Lecturer: Prof Peter Norreys

Course weight: 0.75 units/12 lectures

Assessment method: homework completion only

Prerequisites: For MMathPhys students, B3 Quantum Atomic and Molecular Physics. For MSc students, basic atomic physic. The lectures in weeks 1 - 2 of the course reviews the principles of atomic physics from first principles, presented in the B3 course. This is to ensure that students who enrol via the MSc route (external to the University) are brought up to date with those enrolling internally via the Oxford undergraduate physics course.

Course synopsis: Hot plasma is ubiquitous throughout the Universe and first appeared in the epoch of recombination that produced the cosmic background radiation about 378,000 years after the Big Bang. Since then quantum processes, particularly the emission and absorption of electromagnetic radiation from plasma, have provided essential information about the macroscopic structure of matter in the visible Universe. They are key to understanding stellar structure and evolution (along with helioseismology) by providing constraints on radiative transfer associated with nucleosynthesis of chemical elements in stellar interiors and in supernovae explosions. The effort to harness the immense power of nuclear fusion using magnetic or inertial confinement fusion schemes is being actively pursued world-wide. Indeed, these plasmas are among the most intense sources of X-rays in the laboratory and are used to study materials under extreme conditions of density and temperature. Emerging new tools, such as X-ray free electron lasers, are also being applied to these problems for the first time.

This course will introduce the student to the use of quantum mechanics in the computational modelling of hot plasmas. In the first part, an introduction to atomic processes is first provided to remind students of the basic principles of Slater's configurational model and Racah's tensor operator method. Then, the properties of electronic configurations and transition arrays are described, along with how they are used to replace the corresponding sets of individual levels and radiative lines. Following that, we will describe how these are applied to plasma dynamics and atomic processes, along with elegant new methods of super-configurations and effective temperatures. Finally, current applications are described, along with numerical and experimental examples.

MMathPhys/MSc in Mathematical and Theoretical Physics

Hilary Courses

Advanced Fluid Dynamics

Department: Physics

Lecturers: Prof Paul Dellar and Prof Michael Barnes

Course Weight: 1 unit/ 16 lectures

Assessment Method: written exam in week 0 TT or homework completion

General Prerequisites: Basic familiarity with fluid equations and stress tensors as provided, e.g., by Kinetic Theory

Course Synopsis: (Part 1) Low Reynolds number hydrodynamics. The Stokes flow regime, general mathematical results, flow past a sphere. Stresses due to suspended rigid particles. Calculation of the Einstein viscosity for a dilute suspension. Stresses due to Hookean bead-spring dumb-bells. Derivation of the upper convected Maxwell and Oldroyd-B models for viscoelastic fluids. Properties of such fluids. Suspensions of orientable particles, Jeffery's equation, very brief introduction to active suspensions and liquid crystals.

(Part 2) Validity of the MHD approximation. Conservation equations. Magnetic force. Evolution of the magnetic field. MHD waves. Static MHD equilibria. Relaxation. MHD stability (normal modes, energy principle, application to a z-pinch). Non-ideal MHD.

Advanced Quantum Field Theory

Department: Physics

Lecturer: Prof John Wheeler (Lectures 1-12), Prof John March-Russell (lectures 13-24)

Course Weight: 1.5 units/24 lectures

Assessment Method: written exam in TT week 0

Prerequisites: Quantum Field Theory (MT), Groups and Representations (MT)

Course Synopsis:

1. Feynman Path integral and generating functionals
2. Quantising the Abelian gauge field, Fadeev-Popov mechanism and ghost fields
3. Scalar and fermionic QED: tree graph processes
4. QED at one loop: dimensional regularization, BRST, Ward identities, renormalization
5. Introduction to non-abelian gauge theory, gauge fixing, Feynman rules and scattering processes in QCD
6. Renormalization group and effective field theory
7. Spontaneous symmetry breaking. The Higgs mechanism.
8. Introduction to non-perturbative QFT: Basics of confinement and chiral symmetry breaking. Solitons

Sequels: The Standard Model and Beyond 1 & 2 (TT), Conformal Field Theory (TT), Quantum Field Theory in Curved Space-Time (TT)

Algorithms and Computations in Theoretical Physics

Department: Physics

Lecturer: Prof Werner Krauth

Course Weight: 1 unit/16 lectures

Assessment Method: homework completion only

Course Synopsis: This course introduces to algorithms and scientific computing from the viewpoint of statistical physics. It is also a practical, example-based, primer on subjects such as Markov chains, molecular dynamics, phase transitions, path integrals, superfluids and Bose-Einstein condensation, among others. The course stresses rigorous foundations and modern developments in mathematics (mixing, non-reversibility, perfect sampling,...) and physics (entropic phase transition, Kosterlitz-Thouless physics, cold atoms,...), yet is entirely based on short Python programs. It will prepare advanced undergraduates in theoretical physics/math for the requirements of modern-day research, with the huge roles played by algorithmic thinking and by statistics, both with their power, their paradoxes and their intricacies.

For further details see here: [course page](#)

C3.2 Geometric Group Theory

Department: Maths

Lecturer: Prof Panos Papazoglou

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

General Prerequisites: Some familiarity with Cayley graphs, fundamental group and covering spaces (as for example in the course B3.5 Topology & Groups) would be a helpful though not essential prerequisite.

Course Synopsis: The aim of this course is to introduce the fundamental methods and problems of geometric group theory and discuss their relationship to topology and geometry. The first part of the course begins with an introduction to presentations and the list of problems of M. Dehn. It continues with the theory of group actions on trees and the structural study of fundamental groups of graphs of groups. The second part of the course focuses on modern geometric techniques and it provides an introduction to the theory of Gromov hyperbolic groups.

C3.5 Lie Groups

Department: Maths

Lecturer: Prof Pierrick Bousseau

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

General Prerequisites: ASO: Group Theory, A5: Topology and ASO: Multidimensional Analysis and Geometry are all useful but not essential. It would be desirable to have seen notions of derivative of maps from $\mathbb{R}^n$ to $\mathbb{R}^m$, inverse and implicit function theorems, and submanifolds of $\mathbb{R}^n$. Acquaintance with the notion of an abstract manifold would be helpful but not really necessary.

Course Synopsis: The theory of Lie Groups is one of the most beautiful developments of pure mathematics in the twentieth century, with many applications to geometry, theoretical physics and mechanics. The subject is an interplay between geometry, analysis and algebra. Lie groups are groups which are simultaneously manifolds, that is geometric objects where the notion of differentiability makes sense, and the group multiplication and inversion are differentiable maps. The majority of examples of Lie groups are the familiar groups of matrices. The course does not require knowledge of differential geometry: the basic tools needed will be covered within the course.

C3.11 Riemannian Geometry

Department: Maths

Lecturer: Prof Andrew Dancer

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

General Prerequisites: Differentiable Manifolds is required. An understanding of covering spaces will be strongly recommended.

Course Synopsis: Riemannian Geometry is the study of curved spaces and provides an important tool with diverse applications from group theory to general relativity. The surprising power of Riemannian Geometry is that we can use local information to derive global results. This course will study the key notions in Riemannian Geometry: geodesics and curvature. Building on the theory of surfaces in R^3 in the Geometry of Surfaces course, we will describe the notion of Riemannian submanifolds, and study Jacobi fields, which exhibit the interaction between geodesics and curvature. We will prove the Hopf--Rinow theorem, which shows that various notions of completeness are equivalent on Riemannian manifolds, and classify the spaces with constant curvature. The highlight of the course will be to see how curvature influences topology. We will see this by proving the Cartan--Hadamard theorem, Bonnet--Myers theorem and Synge's theorem.

C3.12 Low-dimensional Topology and Knot Theory

Department: Maths

Lecturer: Prof Andras Juhasz

Course Weight: 1 unit/ 16 lectures

Assessment Method: written exam in TT

General Prerequisites: B3.5 Topology and Groups (MT) and C3.1 Algebraic Topology (MT) are essential. We will assume working knowledge of the fundamental group, covering spaces, homotopy, homology, and cohomology. B3.2 Geometry of Surfaces (MT) and C3.3 Differentiable Manifolds (MT) are useful but not essential, though some prior knowledge of smooth manifolds and bundles should make the material more accessible.

Course synopsis: Low-dimensional topology is the study of 3- and 4-manifolds and knots. The classification of manifolds in higher dimensions can be reduced to algebraic topology. These methods fail in dimensions 3 and 4. Dimension 3 is geometric in nature, and techniques from group theory have also been very successful. In dimension 4, gauge-theoretic techniques dominate. This course provides an overview of the rich world of low-dimensional topology that draws on many areas of mathematics. We will explain why higher dimensions are in some sense easier to understand, and review some basic results in 3- and 4-manifold topology and knot theory.

C5.4 Networks

Department: Maths

Lecturer: Prof Peter Grindrod

Course Weight: 1 unit/16 lectures

Assessment Method: mini-project

General prerequisites: Basic notions of linear algebra, probability, dynamical systems, and some computational experience. The student may use the language of their choice for computational experiments. Relevant notions of graph theory will be reviewed and illustrated.

Course Synopsis: Network Science provides generic tools to model and analyse systems in a broad range of disciplines, including biology, computer science and sociology. This course aims at providing an introduction to this interdisciplinary field of research, by integrating tools from graph theory, statistics and dynamical systems. Most of the topics to be considered are active modern research areas. This year the course has been altered to incorporate some new material on dynamically evolving networks and the analysis of scaling properties of growing networks.

C5.6 Applied Complex Variables

Department: Maths

Lecturer: Prof James Oliver

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

General Prerequisites: The course requires second year core analysis (A2 complex analysis). It continues the study of complex variables in the directions suggested by contour integration and conformal mapping. A knowledge of the basic properties of Fourier Transforms is assumed. Part A Waves and Fluids and Part C Perturbation Methods are helpful but not essential.

Course synopsis: The course begins where core second-year complex analysis leaves off, and is devoted to extensions and applications of that material. The solution of Laplace's equation using conformal mapping techniques is extended to general polygonal domains and to free boundary problems. The properties of Cauchy integrals are analysed and applied to mixed boundary value problems and singular integral equations. The Fourier transform is generalised to complex values of the transform variable, and used to solve mixed boundary value problems and integral equations via the Wiener-Hopf method.

C7.4 Intro to Quantum Information

Department: Maths

Lecturer: Prof Artur Ekert

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

General Prerequisites: Quantum Theory. The course material should be of interest to physicists, mathematicians, computer scientists, and engineers. The following will be assumed as prerequisites for this course:

- elementary probability, complex numbers, vectors and matrices;
- Dirac bracket notation;
- a basic knowledge of quantum mechanics especially in the simple context of finite dimensional state spaces (state vectors, composite systems, unitary matrices, Born rule for quantum measurements);
- basic ideas of classical theoretical computer science (complexity theory) would be helpful but are not essential.

Prerequisite notes will be provided giving an account of the necessary material. It would be desirable for you to look through these notes slightly before the start of the course.

Course Synopsis: The classical theory of computation usually does not refer to physics. Pioneers such as Turing, Church, Post and Goedel managed to capture the correct classical theory by intuition alone and, as a result, it is often falsely assumed that its foundations are self-evident and purely abstract. They are not! Computers are

physical objects and computation is a physical process. Hence when we improve our knowledge about physical reality, we may also gain new means of improving our knowledge of computation. From this perspective it should not be very surprising that the discovery of quantum mechanics has changed our understanding of the nature of computation. In this series of lectures you will learn how inherently quantum phenomena, such as quantum interference and quantum entanglement, can make information processing more efficient and more secure, even in the presence of noise.

C7.6 General Relativity II

Department: Maths

Lecturer: Dr Christopher Couzens

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

Prerequisites: C7.5 General Relativity I

Course synopsis: In this, the second course in General Relativity, we have two principal aims. We first aim to increase our mathematical understanding of the theory of relativity and our technical ability to solve problems in it. We apply the theory to a wider class of physical situations, including gravitational waves and black hole solutions. Orbits in the Schwarzschild solution are given a unified treatment which allows a simple account of the three classical tests of Einstein's theory. This leads to a greater understanding of the Schwarzschild solution and an introduction to its rotating counterpart, the Kerr solution. We analyse the extensions of the Schwarzschild solution show how the theory of black holes emerges and exposes the radical consequences of Einstein's theory for space-time structure.

C7.7 Random Matrix Theory

Department: Maths

Lecturer: Prof Louis-Pierre Arguin

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

General Prerequisites: There are no formal prerequisites, but familiarity with basic concepts and results from linear algebra and probability will be assumed, at the level of A0 (Linear Algebra) and A8 (Probability).

Course synopsis: Random Matrix Theory provides generic tools to analyse random linear systems. It plays a central role in a broad range of disciplines and application areas, including complex networks, data science, finance, machine learning, number theory, population dynamics, and quantum physics. Within Mathematics, it connects with asymptotic analysis, combinatorics, integrable systems, numerical analysis, probability, and stochastic analysis. This course aims to provide an introduction to this highly active, interdisciplinary field of research, covering the foundational concepts, methods, questions, and results.

Collisionless Plasma Physics

Department: Physics

Lecturer: Dr Daniel Kennedy and Dr Plamen Ivanov

Course Weight: 1 unit/18 lectures

Assessment Method: take-home exam or homework completion

General Prerequisites: Kinetic Theory (MT), an undergraduate course on Electricity and Magnetism

Course Synopsis:

Part I. Plasma waves:

Cold plasma waves in a magnetised plasma. WKB theory of cold plasma wave propagation in an inhomogeneous plasma, cut-offs and resonances. Hot plasma waves in a magnetised plasma. Cyclotron resonance.

Part II. Kinetics of strongly magnetised plasmas:

Kinetic description of a collisionless, magnetised plasma; kinetic MHD. Barnes damping, firehose and mirror instabilities. Particle motion. Drift kinetics. Drift waves and the ion-temperature-gradient instability. Electron drift kinetics (time permitting): kinetic Alfvén waves, electron-temperature-gradient instabilities.

Cosmology

Department: Physics

Lecturer: Dr David Alonso

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

Prerequisites: General Relativity I (MT) or equivalent.

Einstein field equations and the Friedman equations, universe models, statistics of expanding background, relativistic cosmological perturbations, observations, from the Hubble flow to the CMB.

Galactic and Planetary Dynamics

Department: Physics

Lecturer: Prof John Magorrian

Course Weight: 1 unit/16 lectures

Assessment Method: TBC

Prerequisites: Kinetic Theory (MT)

Course Synopsis: Review of Hamiltonian mechanics. Orbit integration. Classification of orbits and integrability. Construction of angle-action variables. Hamiltonian perturbation theory. Simple examples of its application to the evolution of planetary and stellar orbits. Methods for constructing equilibrium galaxy models. Applications. Fundamentals of N-body simulation. Dynamical evolution of isolated galaxies. Interactions with companions.

Geophysical Fluid Dynamics

Department: Physics

Lecturer: Prof Tim Woollings

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in TT

Course Synopsis: Rotating frames of reference. Geostrophic and hydrostatic balance. Pressure coordinates. Shallow water and reduced gravity models, f and β -planes, potential vorticity. Inertia-gravity waves, dispersion relation, phase and group velocity. Rossby number, equations for nearly geostrophic motion, Rossby waves, Kelvin waves. Linearised equations for a stratified, incompressible fluid, internal gravity waves, vertical modes. Quasigeostrophic approximation: potential vorticity equation, Rossby waves, vertical propagation and trapping. Eady model of

baroclinic instability. Overview of large-scale structure and circulation of atmospheres and oceans, poleward heat transport. Angular momentum and Held-Hou model of Hadley circulations. Applications to Mars and slowly-rotating planets. Tide-locked exoplanets. Giant planets: Multiple jets, stable eddies and free modes.

High Energy Density Plasma Physics

Department: Physics

Lecturer: Prof Peter Norreys and Dr Ramy Aboushelbaya

Course Weight: 1 unit/16 lectures

Assessment Method: homework completion only

Course Synopsis: In this course, the topics will be introduced for first principles. The student will be taken through the fundamental physics of laser energy absorption in matter up to and including the new laser QED plasma regime at extreme intensities. The student will be introduced to hydrodynamic motion via first principles derivation of the Navier-Stokes equations as well as compression and rarefaction waves. Then a thorough grounding in hydrodynamic instabilities will be provided, including the Rayleigh-Taylor instability and the applications of linear theory. This will be followed by the extension to the convective instability; mode coupling; the Kelvin-Helmholtz; shock stability and the Richtmyer-Meshkov instability. The behaviour of shock waves in one dimension will then be discussed, including the derivation of the Rankine-Hugoniot equations; the effects of boundaries and interfaces; blast waves and shocks in solids. Following that, the physics of convergent shocks will be described. These include homogeneous expansion/contraction self-similar flows as well as shock dynamics. The hydrodynamic behaviour is governed by the equations of state including thermodynamic properties, so the student will be introduced to equations of state for gases, plasmas, solids and liquids. For thermal energy transport, the thermal energy transport equation is derived, as are the effects of the conductivity coefficients, inhibited thermal transport, electron-ion energy exchange, before electron degeneracy effects are introduced. The physics of radiation energy transport will be described, including radiation as a fluid and the Planck distribution function; radiation flux definition; solutions to the radiation energy transfer equations; material opacities; non-LTE radiation transport; radiation dominated hydrodynamics. Finally, dimensionless scaling laws for hydrodynamics will be outlined, ones that provide the student with a link between the fascinating detailed microphysics of laboratory plasma phenomena and exquisite astrophysical observations.

Nonequilibrium Statistical Physics

Department: Physics

Lecturer: Prof Ramin Golestanian

Course Weight: 1 unit/16 lectures

Assessment Method: TBC

Course Synopsis: Stochastic Langevin dynamics. Brownian motion. Nonequilibrium kinetics. Master equation. Fokker-Planck equation. Kramers rate theory and mean first-passage time. Brownian ratchets. Multiplicative noise. Path integral formulation and Martin-Siggia-Rose method. Fluctuation theorems.

Quantum Matter 2: Quantum Fluids

Department: Physics

Lecturer: Prof Sid Parameswaran

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in week 0 TT or homework completion

Course Synopsis: "Quantum fluids" are systems of many interacting particles where the role of quantum statistics is significant, and can lead to macroscopic quantum effects. This course focuses on the simplest examples, ones where Galilean invariance is a good approximation, i.e. the role of crystal lattices or imperfections is ignored. We will first discuss phenomenological and microscopic models of superfluids of bosons (such as Helium 4), before discussing the case of charged superfluids. This will lead us naturally into discussions of the Meissner effect and the Anderson-Higgs mechanism. To describe electronic superconductors (or paired fermionic superfluids such as Helium 3) microscopically, we will first need to take a detour through the theory of the interacting electrons gas and Landau's theory of Fermi liquids, before discussing the Bardeen-Cooper-Schrieffer theory. Time permitting, the course will close with a discussion of arguably the most exotic quantum fluids discovered to date: the two-dimensional quantum Hall liquids that form out of electron gases placed in high magnetic fields, which give rise to fractional charge.

Quantum Matter 3: Quantum Dynamics and Information in Many-particle Systems

Department: Physics

Lecturer: Prof Fabian Essler

Course Weight: 1 unit/16 lectures (continues in Trinity term)

Assessment Method: Homework completion only

Course Synopsis:

- 1 Elements of Quantum Statistical Mechanics
 - 1.1 Pure and Mixed States, (Reduced) Density Matrices
 - 1.2 Entropy, Ensembles and Typicality
- 2 Eigenstates of Local Many-Particle Hamiltonians
 - 2.1 Tight-Binding Model of Spinless Fermions
 - 2.2 Entanglement Measures
 - 2.3 Entanglement Entropy of Energy Eigenstates
 - 2.4 The Spin-1 AKLT Chain
 - 2.5 Matrix Product State Methods
 - 2.6 Symmetry Protected Topological Order
- 3 Quantum Many-Particle Dynamics
 - 3.1 Quantum Quenches
 - 3.2 (Generalized) Thermalization
 - 3.3 Eigenstate Thermalization Hypothesis
 - 3.4 BBGKY Hierarchy
 - 3.5 Self-Consistent Time-Dependent Mean-Field Approximation
 - 3.6 Quantum Boltzmann Equation
- 4 Open and Driven Quantum Systems
 - 4.1 Quantum Master Equations
 - 4.2 Periodically Driven Systems And Quantum Circuits

String Theory I

Department: Maths

Lecturer: Prof Xenia de la Ossa

Course Weight: 1 unit/16 lectures

Assessment Method: TBC

Prerequisites: Quantum Field Theory (MT)

Course Synopsis: Historical background, Dolen-Horn-Schmid duality, the Veneziano and Virasoro-Shapiro amplitudes. Nambu-Goto and Polyakov world-sheet actions, equations of motion and constraints, open and closed strings and their corresponding boundary conditions. Old covariant quantization: the Virasoro algebra, physical state conditions, ghosts, critical spacetime dimension, and spacetime particle spectrum. Basic considerations of light-cone gauge quantization. Vertex operators and string scattering amplitudes. Strings in background fields, spacetime effective action. Circle compactification, elementary consideration of D-branes, T-duality.

Supersymmetry and Supergravity

Department: Maths

Lecturer: Dr Michele Levi

Course Weight: 1 unit/16 lectures

Assessment Method: written exam in week 0 TT

Course Synopsis:

1. Context and Motivation.
2. Spinors Preliminary.
3. Supersymmetry Algebra.
4. Superspace and Superfields.
5. Chiral Superfields and Supersymmetric Actions.
6. Supersymmetric Gauge Theories.
7. Spontaneous Symmetry Breaking.

MMathPhys/MSc in Mathematical and Theoretical Physics

Trinity Courses

Advanced Topics in Plasma Physics

Department: Physics

Lecturer: Dr Daniel Kennedy

Course Weight: 0.75 units/12 lectures

Assessment Method: homework completion only

Course Synopsis: Basics of magnetic-confinement fusion. Magnetic geometry and flux surfaces in toroidal devices. Equilibrium vs fluctuations. Scale separation in time and space.

Asymptotic expansion the Vlasov-Landau equation. Gyrokinetic variables and gyroaverages. Derivation of the gyrokinetic equilibrium. Equilibrium Maxwell's equations. Derivation of the gyrokinetic equation for plasma fluctuations. Fluctuating Maxwell's equations. Free-energy conservation in gyrokinetics. Plasma instabilities. Linear gyrokinetic theory and temperature-gradient-driven instabilities.

Astroparticle Physics

Department: Physics

Lecturer: Prof Joseph Conlon

Course Weight: 1 unit/16 lectures

Assessment Method: homework completion only

Pre-requisites: Quantum Field Theory (MT), General Relativity I (MT)

Course synopsis: The Universe observed, constructing world models, reconstructing our thermal history, decoupling of the cosmic microwave background, primordial nucleosynthesis. Dark matter: astrophysical phenomenology, relic particles, direct and indirect detection. Cosmic particle accelerators, cosmic ray propagation in the Galaxy. The energy frontier: ultrahigh energy cosmic rays and neutrinos. The early Universe: constraints on new physics, baryo/leptogenesis, inflation, the formation of large-scale structure, dark energy.

Collisional Plasma Physics

Department: Physics

Lecturer: Prof Alex Schekochihin

Course Weight: 1 unit/16 lectures

Assessment Method: homework completion only

Prerequisites: Kinetic Theory (MT), Advanced Fluid Dynamics (HT), Collisionless Plasma Physics (HT)

Course Synopsis: Collision operators: Fokker-Plank collision operator, conservation properties, entropy, electron-ion and ion-electron collisions, linearized collision operator. Collisional transport (Braginskii equations: derivation of Spitzer resistivity and electron heat conduction, ion heat conduction and viscosity. Resistive MHD: tearing modes, magnetic reconnection. Introduction to tokamak theory: Pfirsch-Schlueter collision transport regime for electrons.

Conformal Field Theory

Department: Maths

Lecturer: Prof Robin Karlsson

Course Weight: 1 unit/16 hours

Assessment Method: homework completion only

Prerequisites: Quantum Field Theory (MT)

Course Synopsis:

- Motivation: RG flows and scale invariance.
- Conformal transformations.
- Consequence of conformal invariance.
- Radial quantization and the operator algebra.
- Conformal invariance in two dimensions.
- The Virasoro algebra.
- Minimal models.
- Conformal bootstrap in $d > 2$.

Machine Learning Fundamentals with Applications to Physics and Mathematics

Department: Physics

Lecturer: Dr Andrei Constantin

Course Weight: 1 unit/16 lectures

Assessment Method: homework completion only

Prerequisites: Prior exposure to Mathematica and Python will be helpful, but not mandatory.

Course Synopsis: Over the past five to ten years, machine learning and artificial intelligence in general, have evolved into indispensable research tools. This course seeks to offer a comprehensive introduction to the diverse array of machine learning techniques. These methods share the common goal of crafting algorithms that enable computers to make predictions and decisions autonomously, without relying on explicit, handcrafted rules. Using these techniques, one can extract valuable insights from computers that surpass the information initially provided. The course will discuss fundamental principles, algorithms, and a number of applications in Mathematics and Physics, including state reconstruction in quantum physics, model building in particle physics and cosmology, applications to string theory compactifications, conformal bootstrap and knot theory.

Quantum Field Theory in Curved Space-Time

Department: Maths

Lecturer: Dr Pieter Bomans

Course Weight: 1 unit/16 lectures

Assessment Method: homework completion only

Prerequisites: Quantum Field Theory (MT), General Relativity I (MT) and General Relativity II (HT). Advanced Quantum Field Theory and a course on differential geometry will be helpful but not essential.

Course Synopsis: This course builds on the courses in quantum field theory and general relativity. It will focus on classical and quantum aspects of fields in curved space-time. The course will consist of the following topics: Classical fields in curved space, Quantization in curved space, Quantum fields in (Anti) de Sitter space, Quantum fields in Rindler space and the Unruh effect, Hawking radiation, Black hole thermodynamics and the Hawking-Page phase transition, Interactions in curved space-time, Quantum field theory and cosmology.

Quantum Matter 4: Renormalization and Bosonization

Department: Physics

Lecturer: Prof Shivaji Sondhi

Course Weight: 1 unit/16 lectures

Assessment Method: homework completion only

Prerequisites: Although the lectures will be self-contained, this course is designed as a follow-on to Quantum Matter II. Familiarity with ideas introduced in Advanced Quantum Theory and Renormalization Group will be useful but not essential.

Course synopsis: Modern condensed matter physics is increasingly focused on understanding the properties of strongly interacting systems. Traditional techniques that rely on diagrammatic perturbation theory about the independent electron approximation are often insufficient to provide an adequate description of the rich phenomena possible in this setting. Instead, their study requires a variety of ideas often also invoked in the study of quantum field theories in the non-perturbative regime. This course will cover two of these ideas: the renormalization group and (abelian) bosonization.

Renormalization group for Interacting Fermions: momentum-shell RG for φ^4 theory; RG and the Fermi surface; BCS and CDW as competing instabilities; RG in $d=1$ and emergence of Luttinger liquids

Bosonization: Fermion-boson dictionary; application to spinless fermions; sine-Gordon model and Kosterlitz-Thouless flow; emergence of insulators from commensuration

Renormalisation Group

Department: Maths

Lecturer: Prof Fernando Alday

Course Weight: 1 unit/16 lectures

Assessment Method: homework completion only

Prerequisites: Quantum Field Theory (MT), C5.3 Statistical Mechanics (HT) or equivalent

Course Synopsis: This course introduces ideas of scale-invariance and the renormalisation group in statistical physics, using simple lattice models and field theories as examples. Topics include: Real space RG; Fixed points, scaling operators, operator product expansion etc.; Landau Ginsburg theory; Mean field theory; Large N approximation; the 4-epsilon expansion; the 2+epsilon expansion; the Kosterlitz-Thouless transition; The Sine-Gordon model; XY duality.

String Theory II

Department: Maths

Lecturer: Prof Xenia de la Ossa

Course Weight: 1 unit/16 lectures

Assessment Method: homework completion only

General Prerequisites: Quantum Field Theory (MT), String Theory I (HT) , Advanced Quantum Field Theory (HT), Supersymmetry and Supergravity (HT)

Course Synopsis: Classical superstring action, RNS string, quantization and GSO-projection; 10d superstrings: Type IIA, IIB, I and Heterotic strings; Open strings and D-branes; Supergravities and spacetime effective actions, M-theory and 11d supergravity; Compactifications; Dualities between string theories.

The Standard Model and Beyond I

Department: Physics

Lecturer: TBC

Course Weight: 1 unit/ 16 lectures

Assessment Method: homework completion only

Prerequisite: Advanced Quantum Field Theory (HT)

Course Synopsis: Basics of strong interactions: the peculiarities of asymptotic freedom and the uniqueness of gauge theories. Low-energy effective actions: from QCD to the chiral Lagrangian, and Effective Field Theories. Building the Electroweak sector of the Standard Model. Exploring the structure of the Electroweak sector. QCD at colliders [if time permits]: OPE and factorisation, from hadrons to partons
You may find the following textbooks useful: H. Georgi, Weak Interactions and Modern Particle Theory; J.F. Donoghue, E. Golowich, Barry R. Holstein, Dynamics of the standard model.

The Standard Model and Beyond II

Department: Physics

Lecturer: Prof John March-Russell

Course Weight: 1 unit/16 lectures

Assessment Method: homework completion only

Prerequisite: Advanced Quantum Field Theory (HT)

Topics in Soft and Activer Matter Physics

Department: Physics

Lecturer: Prof Ard Louis

Course Weight: 0.5 units/8 lectures

Assessment Method: homework completion only

Prerequisites: Advanced Fluid Dynamics (HT)

Course Synopsis:

This is a reading course. Under the guidance of the course organiser, students will give presentations based on key papers in soft condensed matter theory. Some examples of the topics for these presentations are: Active nematics and active gels. Wetting, spreading and contact line dynamics. Hydrodynamics of microswimmers: Stokes equation, scallop theorem, multipole expansion, active suspensions. Fluctuations and response.